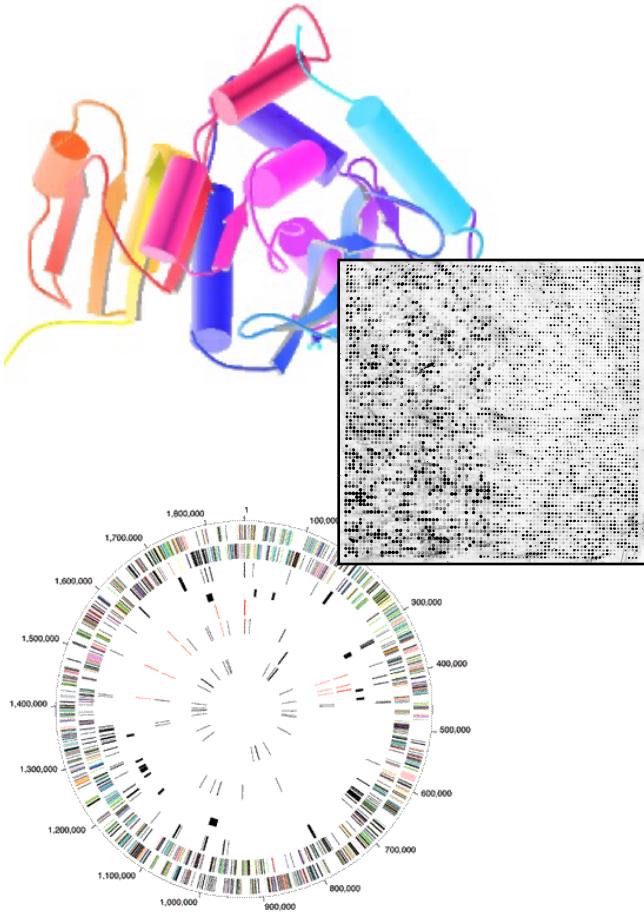


# Biomed. Data Sci. Multiple Sequences



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(Last edit in spring '20)

# Multiple Sequence Alignment Topics

- Multiple Sequence Alignment
- Motifs
  - Fast identification methods
- Profile Patterns
  - Refinement via EM
  - Gibbs Sampling
- HMMs
- Applications
  - Protein Domain databases
  - Regression vs expression

- One of the most essential tools in molecular biology

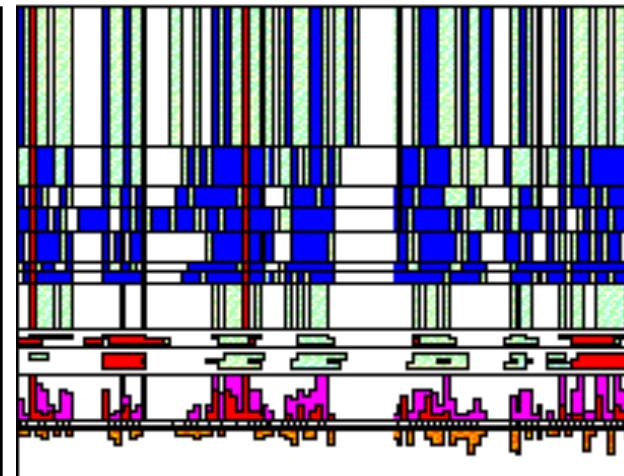
It is widely used in:

- Phylogenetic analysis
- Prediction of protein secondary/tertiary structure
- Finding diagnostic patterns to characterize protein families
- Detecting new homologies between new genes and established sequence families

AGRI_CHICK	154	CVCAPAS.	GS...GVA.ESI	VCGSDGKDYRS	BQDILNKHAC	.DK.	.QENVFKKF	DGAC	201
AGRI_RAT	165	CLCPPT.	CF...GAp.DGT	VCGSDGVYF	SECQLLSHAC	.AS.	.QEHI	FKKF	GAC
FSA_HUMAN	116	CVCAPD.	GS...NI	twKGPVCCLDGKTYRNECALLKARC	.KE.	.QPELEVQY	QCRC	164	
FSA_PIG	116	CVCAPD.	GS...NI	twKGPVCCLDGKTYRNECALLKARC	.KE.	.QPELEVQY	QCRC	164	
FSA_RAT	116	CVCAPD.	GS...NI	twKGPVCCLDGKTYRNECALLKARC	.KE.	.QPELEVQY	QCRC	164	
FSA_SHEEP	109	CVCAPD.	GS...NI	twKGPVCCLDGKTYRNECALLKARC	.KE.	.QPELEVQY	QCRC	157	
IAC1_BOVIN	14	CKVYTEA.	CT...RE.	YNPIQDSAATYSNEECTF	.CNEKM.NN.	.DADIHF	NHFGEC	61	
IAC2_BOVIN	7	CAEFKDP.	KVYCT.	RE.SNPHCGSNGETYGNKQAF	.GKAVM.KS.	.GGKINL	KHKGC	57	
IACA_PIG	7	CNVYRSH.	LFFCT.	RQ.MDPICCGINGKSYANPCIF	.GSEKG.LR.	.NQKFD	FGHWHGC	57	
IACS_PIG	12	CDVYRSH.	LFFCT.	RE.MDPICCGINGKSYANPCIF	.GSEKL.GR.	.NEKFD	FGHWHGC	62	
IAC_MACFA	33	CARYQLPG.	CP...RD.	PNPVGCGIDMITYPNBGTI	.GKMKIR.ES.	.GQNIIKIL	RRCFC	81	
IOV7_CHICK	94	CSPYLQVRDGNtMVACI.	RI..L	KPVCCSDSFTYDNPCGFI	.CAYNA.EH.	.HTNISLKH	DGEC	150	
IOVO_ABUP1	8	CSDPHPKP.	ACI.	QE.QKPLCGSDNKTYDNKGSF.	.CNAVVS.DS.	.NGTTLT	LSHFGC	56	
IOVO_ALECH	6	CSEYPKP.	ACT.	LE.YRPLLCGSDSKTYGNKQNF	.CNAVVS.ES.	.NGTTLT	LSHFGC	54	
IPSG_VULVU	68	CTEYSDM.	CT..MD.	YRPLLCGSDKTYGNKQNF	.CNAVVS.ES.	.RGTFI	FLAKHGC	115	
IPST_ANGAN	12	GERMSAMHA.	CP..MN.	FAPVCGIDGNTYPNBGTI	.GFORQ.NT.	.KTDLIT	KDDRC	61	
IPST_BOVIN	9	CTNEVNG.	CP..RI.	YNPVCCIDGVTYSNECQLL	.CMENK.ER.	.QTPVLI	QKSGFC	56	
IPST_PIG	9	CTSEVSG.	CP..KI.	YNPVCCIDGVTYSNECQLL	.QSENK.KR.	.QTPVLI	QKSGFC	56	
IPST_SHEEP	9	CTNEVNG.	CP..RI.	YNPVCCIDGVTYANBOLL	.CMENK.ER.	.QTPVLI	QKSGFC	56	
OATP_HUMAN	439	CNVDCN.	CP...KI.	WDPVCCNGNLSYLSACLA	.GC..ET.SI.	.GTGINMV	FQNCs	485	
OATP_RAT	439	CNTRCS.	CS...TNT.	WDPVCCENGVAYMSACLA	.GKFKFV.GT.	.GTNM.V	VFDCCS	486	
PE60_PIG	37	CEHMTESPD.	CS...RI.	YDPVCCIDGVTYSECKL	.GLARI.EN.	.KQDIQIV	KFEG	86	
PGT_RAT	444	CRRDCS.	CP...DSF.	FHPVCCENGVEYVFSOHA.	.GC...SS.	.TTNTS	SEASKEPI	488	
PSG1_MOUSE	33	CHDAVAG.	CP...RI.	YDPVCCIDGITYANBOLL	.GFENR.KR.	.IEPV	LIRKGFC	80	
QR1_COTJA	466	CIGQDPA.	ACI..tKD.	YKRVCCIDNNTYDGTGFKQLEGtKM	.GRQLHD	YDNY	GAC	521	
SC1_RAT	424	CVCQDPET.	CP..aKI.	LDQACGIDNQTYASSCHLFATKQMLEGtKK	.GHOLQ	LHDY	FEAGC	479	
SPRC_BOVIN	93	CVCQDP.TS.	Cap.iGE.	FEKVCSDNNTKTFDSSCHFFATKQMLEGtKK	.GHKLH	LHDY	ICFC	149	
SPRC_CAAEL	74	CECISK.	CPeldgDP.	MDKVCAANNQTFISQCDLYRERGLCKR.KSkecska	fNAVKHLEY	GFC	135		
SPRC_MOUSE	92	CVCQDP.TS.	Cap.iGE.	FEKVCSDNNTKTFDSSCHFFATKQMLEGtKK	.GHKLH	LHDY	ICFC	148	
SPRC_XENLA	90	CVCQDPST.	CPts.vGE.	FEKICGIDNNTYDSSCHFFATKQMLEGtKK	.GHKLH	LHDY	ICFC	146	

# Multiple Sequence Alignments

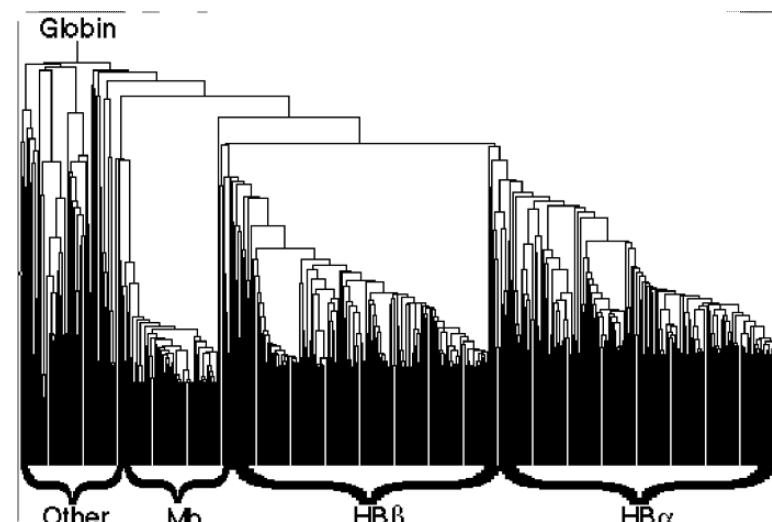
- Practically useful methods only since 1987
- Before 1987 they were constructed by hand
- The basic problem: no dynamic programming approach can be used
- First useful approach by D. Sankoff (1987) based on phylogenetics



(LEFT, adapted from Sonhammer et al. (1997). "Pfam," Proteins 28:405-20. ABOVE, G Barton AMAS web page)

# Progressive Multiple Alignments

- Most multiple alignments based on this approach
- Initial guess for a phylogenetic tree based on pairwise alignments
- Built progressively starting with most closely related sequences
- Follows branching order in phylogenetic tree
- Sufficiently fast
- Sensitive
- Algorithmically heuristic, no mathematical property associated with the alignment
- Biologically sound, it is common to derive alignments which are impossible to improve by eye

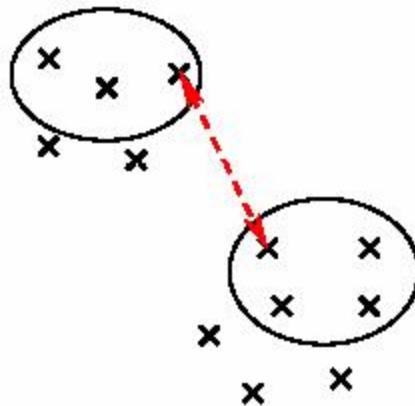


(adapted from Sonhammer et al. (1997). "Pfam," Proteins 28:405-20)

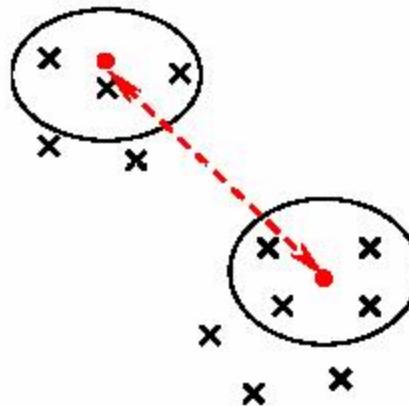
# Clustering approaches for multiple sequence alignment

- Clustal uses average linkage clustering
  - ◊ also called UPGMA  
Unweighted Pair Group Method with Arithmetic mean

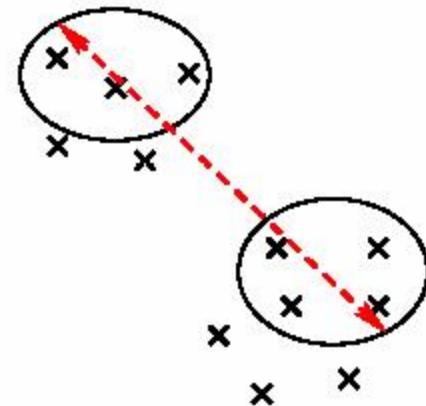
- Simple linkage



- Average linkage



- Complete linkage



<http://compbio.pbworks.com/f/linkages.JPG>

# Problems with Progressive Alignments

- Local Minimum Problem
  - Parameter Choice Problem

## 1. Local Minimum Problem

- It stems from greedy nature of alignment  
(mistakes made early in alignment cannot be corrected later)
- A better tree gives a better alignment  
(UPGMA neighbour-joining tree method)

## 2. Parameter Choice Problem

- - It stems from using just one set of parameters  
(and hoping that they will do for all)

# Domain Problem in Multiple Alignment

Match 3



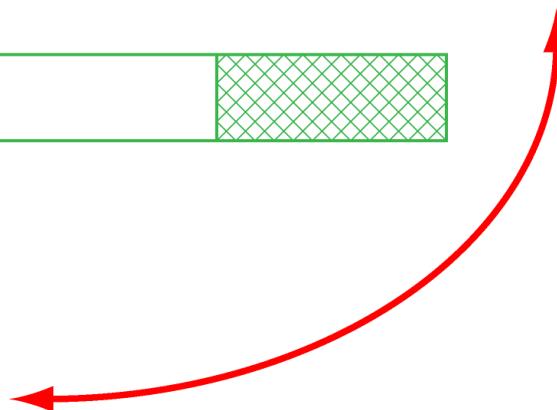
Match 2



Match 1



Query

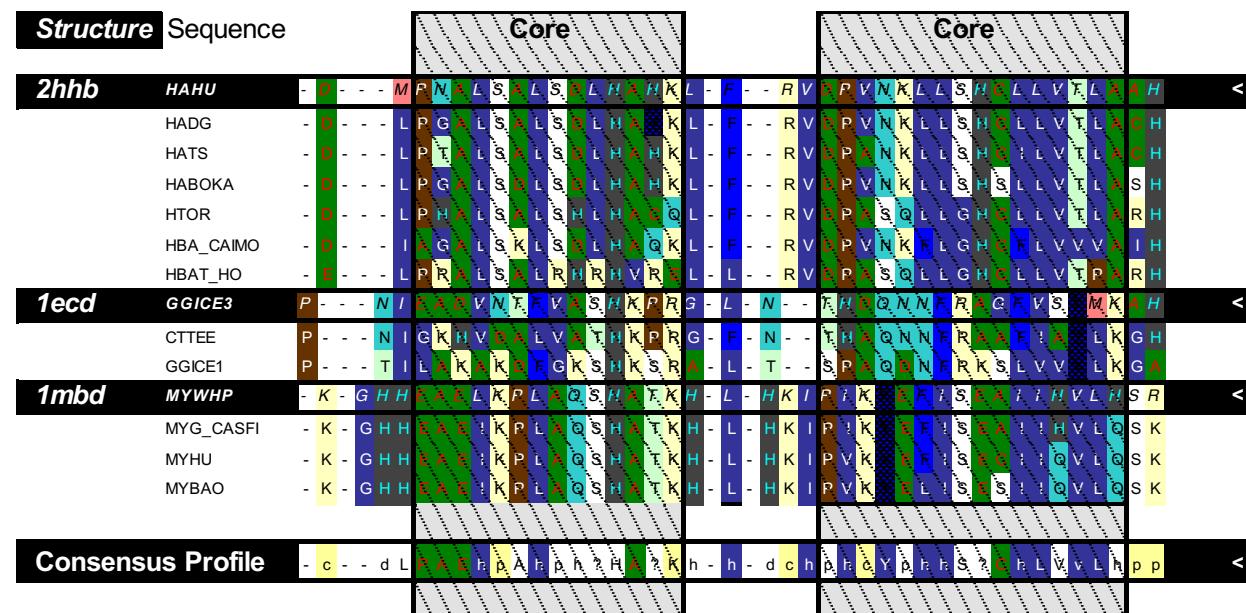


Fuse multiple alignment into:

- **Motif**: a short signature pattern identified in the conserved region of the multiple alignment
- **Profile**: frequency of each amino acid at each position is estimated
- **HMM**: Hidden Markov Model, a generalized profile in rigorous mathematical terms

# Profiles Motifs HMMs

Can get more sensitive searches with these multiple alignment representations (Run the profile against the DB.)



# **Multiple Alignment**

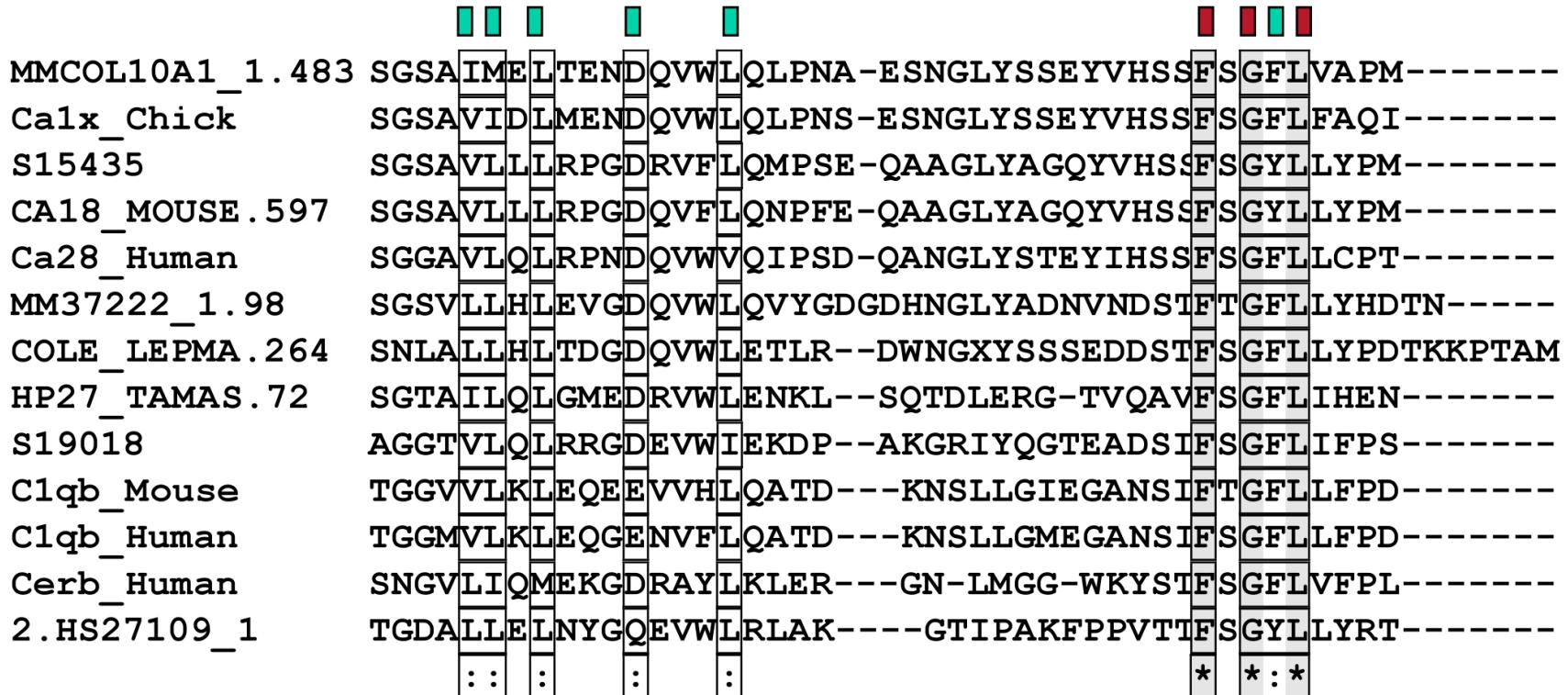
**MOTIFS**

## 2 different applications for motif analysis

- Given a collection of binding sites (or protein sequences with binding motifs), develop a representation of those sites that can be used to search new sites and reliably predict where additional binding sites occur.
- Given a set of sequences known to contain binding sites for a common factor, but not knowing where the sites are, discover the location of the sites in each sequence and a representation of the protein.

# Motifs

- several proteins are grouped together by similarity searches
- they share a conserved motif
- motif is stringent enough to retrieve the family members from the complete protein database
- PROSITE: a collection of motifs (1135 different motifs)



# Prosite Pattern -- EGF like pattern

A sequence of about thirty to forty amino-acid residues long found in the sequence of epidermal growth factor (EGF) has been shown [1 to 6] to be present, in a more or less conserved form, in a large number of other, mostly animal proteins. The proteins currently known to contain one or more copies of an EGF-like pattern are listed below.

- Bone morphogenic protein 1 (BMP-1), a protein which induces cartilage and bone formation.
- *Caenorhabditis elegans* developmental proteins lin-12 (13 copies) and glp-1 (10 copies).
- Calcium-dependent serine proteinase (CASP) which degrades the extracellular matrix proteins type ...
- Cell surface antigen 114/A10 (3 copies).
- Cell surface glycoprotein complex transmembrane subunit .
- Coagulation associated proteins C, Z (2 copies) and S (4 copies).
- Coagulation factors VII, IX, X and XII (2 copies).
- Complement C1r/C1s components (1 copy).
- Complement-activating component of Ra-reactive factor (RARF) (1 copy).
- Complement components C6, C7, C8 alpha and beta chains, and C9 (1 copy).
- Epidermal growth factor precursor (7-9 copies).

The diagram shows a sequence of amino acids represented by 'x' characters. Specific positions are highlighted with colored characters: red 'C's, green 'G', blue 'a', and black 'x'. Vertical lines above and below the sequence indicate positions of identical residues. The sequence is as follows:

x(4)-C-x(0,48)-C-x(3,12)-C-x(1,70)-C-x(1,6)-C-x(2)-G-a-x(0,21)-G-x(2)-C-x  
| | \* \*\*\*\*\*  
+-----+ |

'C': conserved cysteine involved in a disulfide bond.

'G': often conserved glycine

'a': often conserved aromatic amino acid

'\*': position of both patterns.

'x': any residue

-Consensus pattern: C-x-C-x(5)-G-x(2)-C

[The 3 C's are involved in disulfide bonds]

# **Multiple Alignment**

## **PROFILES**

# Profiles

2hhb Human Alpha Hemoglobin		R	V	D	C	V	A	Y	K	
HAHU		R	V	D	C	V	A	Y	K	100
HADG		R	V	D	C	V	A	Y	K	89
HTOR		R	V	D	C	A	A	Y	Q	76
HBA_CAIMO		R	V	D	P	V	A	Y	K	73
HBAT_HORSE		R	V	D	P	A	A	Y	Q	62

1mbd Whale Myoglobin		A	I	C	A	P	A	Y	E	
MYWHP		A	I	C	A	P	A	Y	E	100
MYG_CASFI		R	I	C	A	P	A	Y	E	85
MYHÜ		R	I	C	V	C	A	Y	D	75
MYBAO		R	I	C	V	C	A	Y	D	71

Eisenberg Profile Freq. A

Eisenberg Profile Freq. C

:

Eisenberg Profile Freq. V

Eisenberg Profile Freq. Y

1	0	0	2	2	9	0	0	
0	0	4	3	2	0	0	0	
.	.	.	.	.	.	.	.	
0	5	0	2	3	0	0	0	
0	0	0	0	0	0	0	9	0

↑  
Identity

Consensus = Most Typical A.A.

R V D C V A Y E

Better Consensus = Freq. Pattern (PCA)

R iv cd š š A Y μ

š = (A,2V,C,P);  $\mu$ =(4K,2Q,3E,2D)

Entropy => Sequence Variability

3 7 7 14 14 0 0 14

Profile : a position-specific scoring matrix composed of 21 columns and N rows (N=length of sequences in multiple alignment)

What happens with gaps?

# EGF Profile Generated for SEARCHWISE

Cons	A	C	D	E	F	G	H	I	K	L	M	N	P	Q	R	S	T	V	W	Y	Gap
V	-1	-2	-9	-5	-13	-18	-2	-5	-2	-7	-4	-3	-5	-1	-3	0	0	-1	-24	-10	100
D	0	-14	-1	-1	-16	-10	0	-12	0	-13	-8	1	-3	0	-2	0	0	-8	-26	-9	100
V	0	-13	-9	-7	-15	-10	-6	-5	-5	-7	-5	-6	-4	-4	-6	-1	0	-1	-27	-14	100
D	0	-20	18	11	-34	0	4	-26	7	-27	-20	15	0	7	4	6	2	-19	-38	-21	100
P	3	-18	1	3	-26	-9	-5	-14	-1	-14	-12	-1	12	1	-4	2	0	-9	-37	-22	100
C	5	115	-32	-30	-8	-20	-13	-11	-28	-15	-9	-18	-31	-24	-22	1	-5	0	-10	-5	100
A	2	-7	-2	-2	-21	-5	-4	-12	-2	-13	-9	0	-1	0	-3	2	1	-7	-30	-17	100
s	2	-12	3	2	-25	0	0	-18	0	-18	-13	4	3	1	-1	7	4	-12	-30	-16	25
n	-1	-15	4	4	-19	-7	3	-16	2	-16	-10	7	-6	3	0	2	0	-11	-23	-10	25
P	0	-18	-7	-6	-17	-11	0	-17	-5	-15	-14	-5	28	-2	-5	0	-1	-13	-26	-9	25
c	5	115	-32	-30	-8	-20	-13	-11	-28	-15	-9	-18	-31	-24	-22	1	-5	0	-10	-5	25
L	-5	-14	-17	-9	0	-25	-5	4	-5	8	8	-12	-14	-1	-5	-7	-5	2	-15	-5	100
N	-4	-16	12	5	-20	0	24	-24	5	-25	-18	25	-10	6	2	4	1	-19	-26	-2	100
g	1	-16	7	1	-35	29	0	-31	-1	-31	-23	12	-10	0	-1	4	-3	-23	-32	-23	50
G	6	-17	0	-7	-49	59	-13	-41	-10	-41	-32	3	-14	-9	-9	5	-9	-29	-39	-38	100
T	3	-10	0	2	-21	-12	-3	-5	1	-11	-5	1	-4	1	-1	6	11	0	-33	-18	100
C	5	115	-32	-30	-8	-20	-13	-11	-28	-15	-9	-18	-31	-24	-22	1	-5	0	-10	-5	100
I	-6	-13	-19	-11	0	-28	-5	8	-4	6	8	-12	-17	-4	-5	-9	-4	6	-12	-1	100
d	-4	-19	8	6	-15	-13	5	-17	0	-16	-12	5	-9	2	-2	-1	-1	-13	-24	-5	31
i	0	-6	-8	-6	-4	-11	-5	3	-5	1	2	-5	-8	-4	-6	-2	0	4	-14	-6	31
g	1	-13	0	0	-20	-3	-3	-12	-3	-13	-8	0	-7	0	-5	2	0	-7	-29	-16	31
L	-5	-11	-20	-14	0	-23	-9	9	-11	8	7	-14	-17	-9	-14	-8	-4	7	-17	-5	100
E	0	-20	14	10	-33	5	0	-25	2	-26	-19	11	-9	4	0	3	0	-19	-34	-22	100
S	3	-13	4	3	-28	3	0	-18	2	-20	-13	6	-6	3	1	6	3	-12	-32	-20	100
Y	-14	-9	-25	-22	31	-34	10	-5	-17	0	-1	-14	-13	-13	-15	-14	-13	-7	17	44	100
T	0	-10	-6	-1	-11	-16	-2	-7	-1	-9	-5	-3	-9	0	-1	1	3	-4	-16	-8	100
C	5	115	-32	-30	-8	-20	-13	-11	-28	-15	-9	-18	-31	-24	-22	1	-5	0	-10	-5	100
P	0	-14	-8	-4	-15	-17	0	-7	-1	-7	-5	-4	6	0	-2	0	1	-3	-26	-10	100
P	1	-18	-3	0	-24	-13	-3	-12	1	-13	-10	-2	15	2	0	2	1	-8	-33	-19	100
G	4	-19	3	-4	-48	53	-11	-40	-7	-40	-31	5	-13	-7	-7	4	-7	-29	-39	-36	100
Y	-22	-6	-35	-31	55	-43	11	-1	-25	6	4	-21	-34	-20	-21	-22	-20	-7	43	63	50
S	1	-9	-3	-1	-14	-7	0	-10	-2	-12	-7	0	-7	0	-4	4	4	-5	-24	-9	100
G	5	-20	1	-8	-52	66	-14	-45	-11	-44	-35	4	-16	-10	-10	4	-11	-33	-40	-40	100
E	2	-20	10	12	-31	-7	0	-19	6	-20	-15	5	4	7	2	4	2	-13	-38	-22	100
R	-5	-17	0	1	-16	-13	8	-16	9	-16	-11	5	-11	7	15	-1	-1	-13	-18	-6	100
C	5	115	-32	-30	-8	-20	-13	-11	-28	-15	-9	-18	-31	-24	-22	1	-5	0	-10	-5	100
E	0	-26	20	25	-34	-5	6	-25	10	-25	-17	9	-4	16	5	3	0	-18	-38	-23	100
T	-4	-11	-13	-8	-1	-21	2	0	-4	-1	0	-6	-14	-3	-5	-4	0	0	-15	0	100
D	0	-18	5	4	-24	-11	-1	-11	2	-14	-9	1	-6	2	0	0	0	-6	-34	-18	100
I	0	-10	-2	-1	-17	-14	-3	-4	-1	-9	-4	0	-11	0	-4	0	2	-1	-29	-14	100
D	-4	-15	-1	-2	-13	-16	-3	-8	-5	-6	-4	-1	-7	-2	-7	-3	-2	-6	-27	-12	100

Cons.  
Cys

2hhb	Human Alpha Hemoglobin	R	V	D	C	V	A	Y	K	
	HAHU	R	V	D	C	V	A	Y	K	100
	HADG	R	V	D	C	V	A	Y	K	89
	HTOR	R	V	D	C	A	A	Y	Q	76
	HBA_CAIMO	R	V	D	P	V	A	Y	K	73
	HBAT_HORSE	R	V	D	P	A	A	Y	Q	62
1mbd	Whale Myoglobin	A	I	C	A	P	A	Y	E	
	MYWHP	A	I	C	A	P	A	Y	E	100
	MYG_CASFI	R	I	C	A	P	A	Y	E	85
	MYHÜ	R	I	C	V	C	A	Y	D	75
	MYBAO	R	I	C	V	C	A	Y	D	71
Eisenberg Profile Freq. A		1	0	0	2	2	9	0	0	
Eisenberg Profile Freq. C		0	0	4	3	2	0	0	0	
:		.	.	.	.	.	.	.	.	
Eisenberg Profile Freq. V		0	5	0	2	3	0	0	0	
Eisenberg Profile Freq. Y		0	0	0	0	0	0	9	0	

↑ Identity

R	V	D	C	V	A	Y	E
R	v	cd	š	š	A	Y	μ

$\check{s} = (A, 2V, C, P); \mu = (4K, 2Q, 3E, 2D)$

3	7	7	14	14	0	0	14
---	---	---	----	----	---	---	----

**M(p,a) = chance of finding amino acid a at position p**

$M_{simp}(p,a)$  = number of times a occurs at p divided by number of sequences

However, what if don't have many sequences in alignment?  $M_{simp}(p,a)$  might be biased. Zeros for rare amino acids. Thus:

$$M_{cplx}(p,a) = \sum_{b=1 \text{ to } 20} M_{simp}(p,b) \times Y(b,a)$$

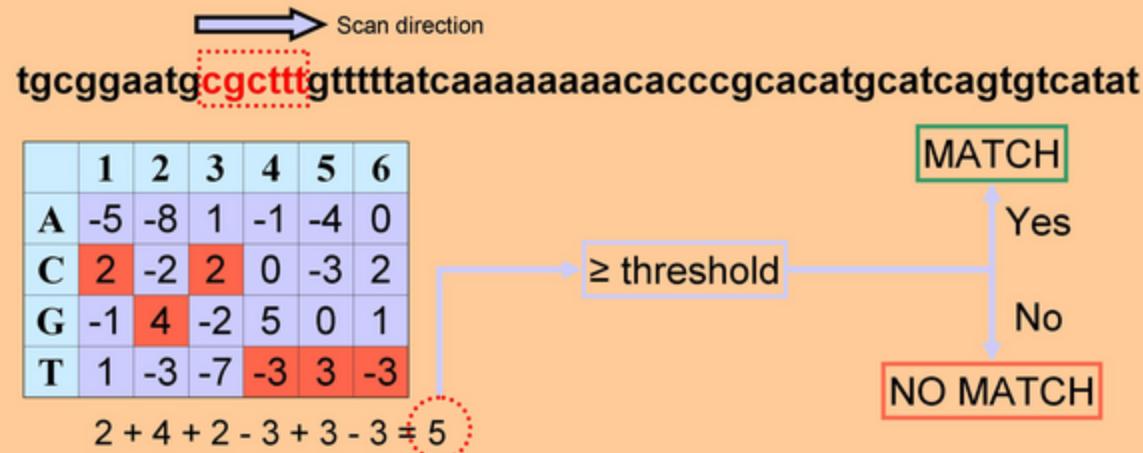
$Y(b,a)$ : Dayhoff matrix for a and b amino acids

$$S(p,a) \sim \sum_{a=1 \text{ to } 20} M_{simp}(p,a) \ln M_{simp}(p,a)$$

# Profiles formula for position $M(p,a)$

# Scanning for Motifs with PWMs

Position Weight Matrices define an additive scheme for scoring sequence. Often, the weights are simply log likelihood ratios of observing a nucleotide in a binding site relative to genomic background. Sequences are scanned by scoring every site, on both the forward and reverse complement strands, and identifying matches as shown in the schematic below:



A particular site is evaluated by adding up the entries from the scoring matrix at each position, and comparing the sum to a match threshold. For log ratio PWMs, an empirically chosen threshold of 60% of the maximum positive score has been used by Harbison et al. and is approximately equal to cutoffs determined by the principled cross-validated method presented in MacIsaac et al. More sophisticated algorithms developed specifically for motif scanning are described briefly in Figure 3.

# Ψ-Blast

Parameters: overall threshold, inclusion threshold, iterations

- Automatically builds profile and then searches with this
- Also PHI-blast

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Nucleic Acids Research, 1997, Vol. 25, No. 17 3389–3402

## Gapped BLAST and PSI-BLAST: a new generation of protein database search programs

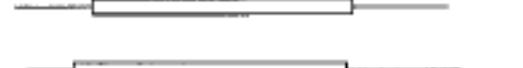
Stephen F. Altschul\*, Thomas J. Madden, Alejandro A. Schäffer<sup>1</sup>, Linghu Zhang,  
Zheng Zhang<sup>2</sup>, Webb Miller<sup>2</sup> and

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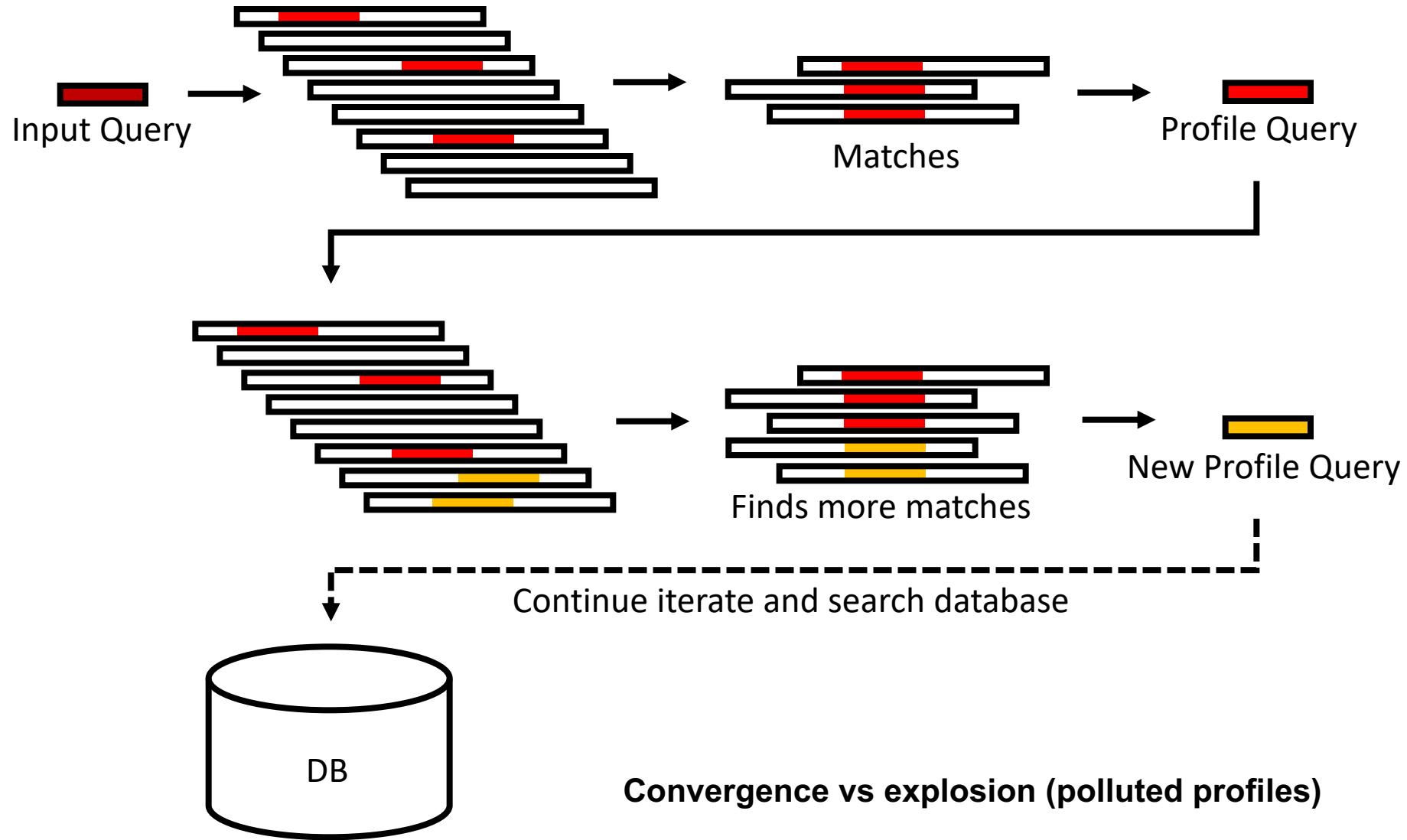
Received June 20, 1997; Revised and Accepted July 17, 1997

### ABSTRACT

The BLAST programs are widely used for quickly searching protein and DNA databases for local similarities. For protein comparison, BLAST uses a heuristic search algorithm that is based on a probabilistic model of local alignments. This paper describes two new programs, Gapped BLAST and PSI-BLAST, that extend the basic BLAST search algorithm to handle gaps and iteratively refine the search to find more distant relationships.

Accession	Alignment	E-value
P49789		
P49779		8e-27
P49775		6e-18
Q11066		3e-07
Q09344		4e-05
P49378		0.001
P32084		0.002

# PSI-BLAST (Position-Specific Iterative Basic Local Alignment Search Tool)



# Low-Complexity Regions

- Low Complexity Regions must be filtered out
  - ◊ Different Statistics for matching  
AAATTAAATTTAAAATTAAATTAAATT  
than  
ACSRPLRVSHRSENCVASNKPQLVKLMTHVKDFCV
  - ◊ Automatic Programs Screen These Out (SEG)
  - ◊ Identify through computation of sequence entropy in a window of a given size  
$$H = \sum f(a) \log_2 f(a)$$
- Also, Compositional Bias
  - ◊ Matching A-rich query to A-rich DB vs. A-poor DB



# Multiple Alignment: Probabilistic Approaches for Determining PWMs

- Expectation Maximization: Search the PWM space randomly
- Gibbs sampling: Search sequence space randomly.

# Expectation-Maximization (EM) algorithm

- Used in statistics for finding maximum likelihood estimates of parameters in probabilistic models, where the model depends on unobserved latent variables.
  - EM alternates between performing
    - an expectation (E) step, which computes an expectation of the likelihood by including the latent variables as if they were observed, and
    - a maximization (M) step, which computes the maximum likelihood estimates of the parameters by maximizing the expected likelihood found on the E step.
  - The parameters found on the M step are then used to begin another E step, and the process is repeated.
1. Guess an initial weight matrix
  2. Use weight matrix to predict instances in the input sequences
  3. Use instances to predict a weight matrix
  4. Repeat 2 [E-step] & 3 [M-step] until satisfied.

Another good source is Wes Craven's 776 course: <https://www.biostat.wisc.edu/~craven/776/lecture9.pdf>

[Adapted from B Noble, GS 541 at UW, <http://noble.gs.washington.edu/~wnoble/genome541/>]

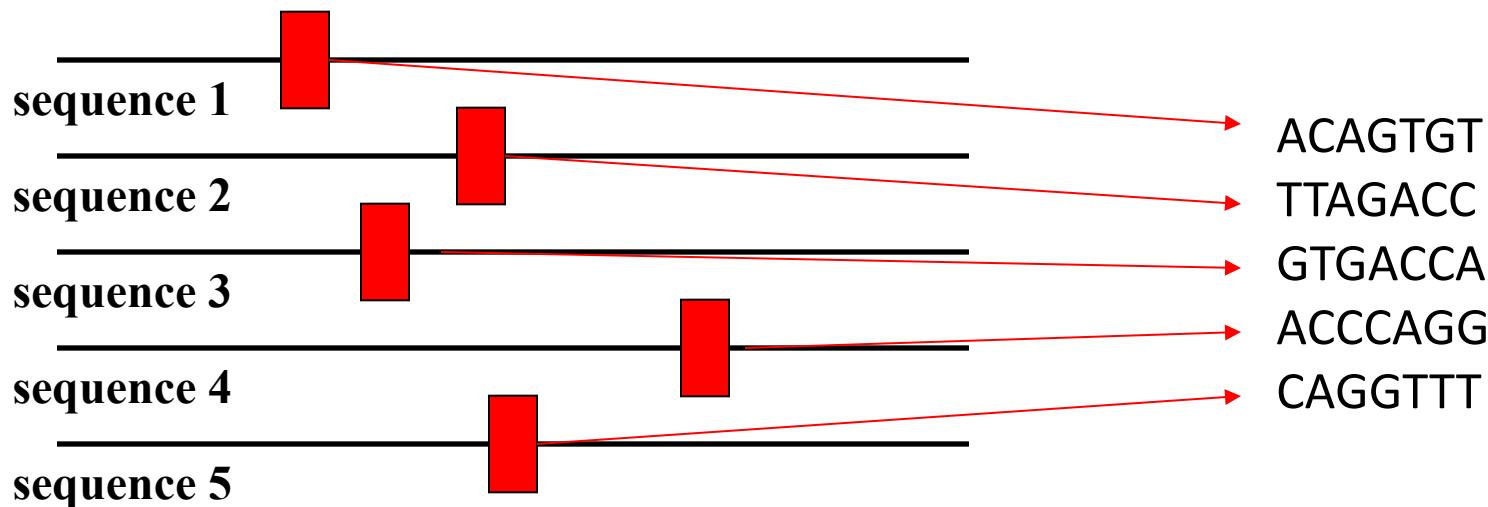
[Also Adapted from C Bruce, CBB752 '09]

# **Multiple Alignment**

## **Gibbs Sampling**

# Initialization

- Step 1: Randomly guess an instance  $s_i$  from each of  $t$  input sequences  $\{S_1, \dots, S_t\}$ .



# Gibbs sampler

- Steps 2 & 3 (search):
  - Throw away an instance  $s_i$ : remaining ( $t - 1$ ) instances define weight matrix.
  - Weight matrix defines instance probability at each position of input string  $S_i$
  - Pick new  $s_i$  according to probability distribution (not necessarily always the  $s_i$  giving the highest prob.)
- Return highest-scoring motif seen

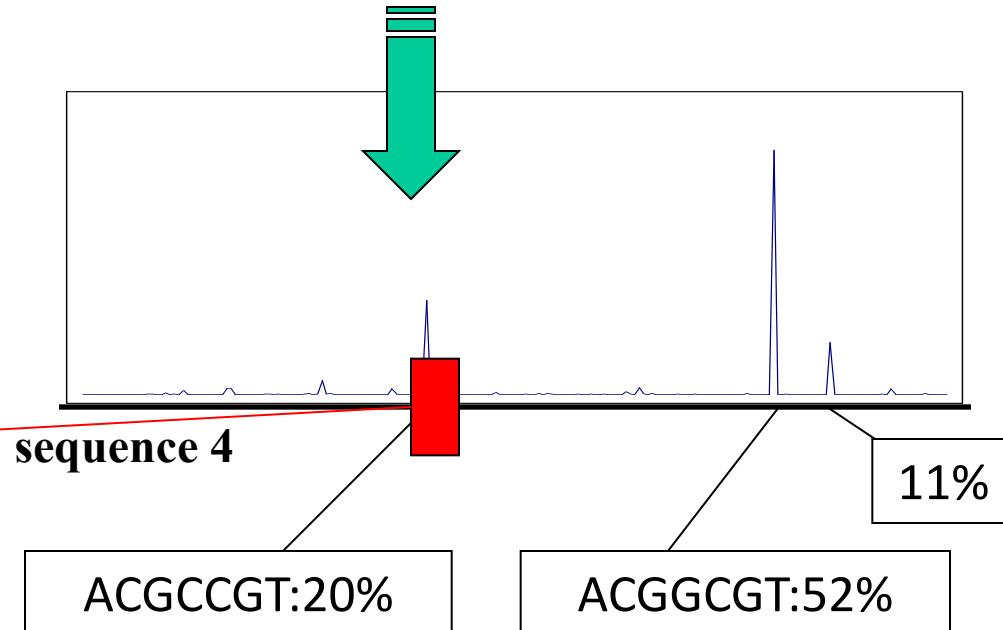
# Sampler step illustration:

ACAGTGT  
TAGGCGT  
ACACCGT  
??????  
CAGGTTT



A	.45	.45	.45	.05	.05	.05	.05
C	.25	.45	.05	.25	.45	.05	.05
G	.05	.05	.45	.65	.05	.65	.05
T	.25	.05	.05	.05	.45	.25	.85

ACAGTGT  
TAGGCGT  
ACACCGT  
**ACGCCGT**  
CAGGTTT



# Comparison

- Both EM and Gibbs sampling involve iterating over two steps
- Convergence:
  - EM converges when the PSSM stops changing.
  - Gibbs sampling runs until you ask it to stop.
- Solution:
  - EM may not find the motif with the highest score.
  - Gibbs sampling will provably find the motif with the highest score, if you let it run long enough.

# **Multiple Alignment**

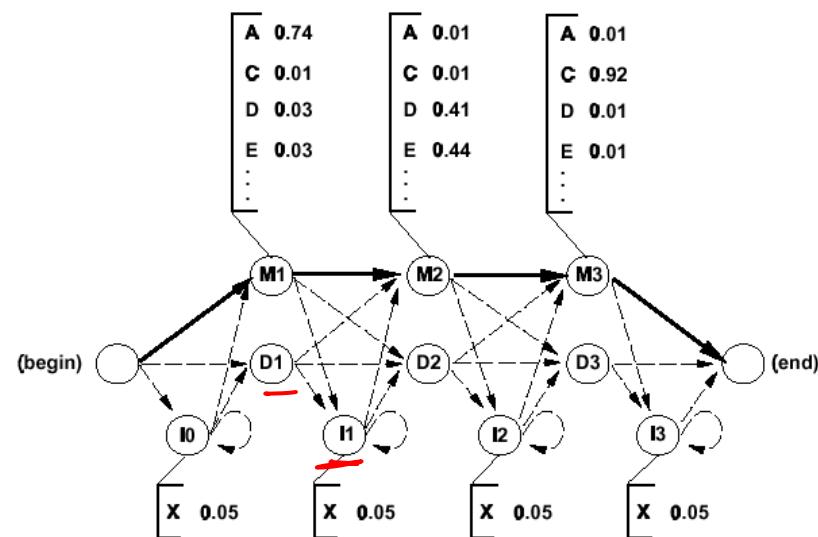
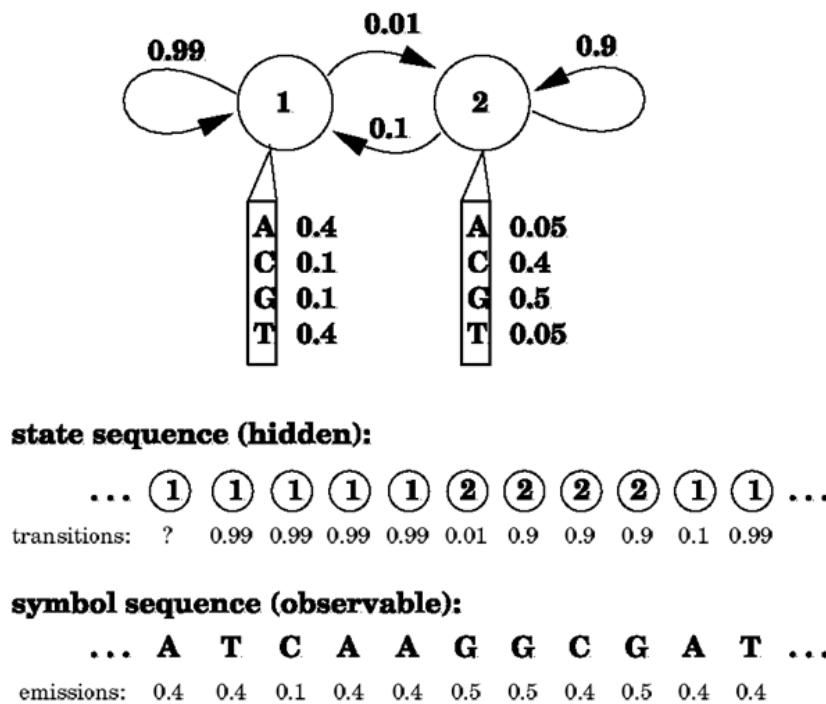
**HMMs**

## Hidden Markov Model:

- a composition of finite number of states,
- each corresponding to a column in a multiple alignment
- each state emits symbols, according to symbol-emission probabilities

## HMMs

Starting from an initial state, a sequence of symbols is generated by moving from state to state until an end state is reached.



(Figures from Eddy, Curr. Opin. Struct. Biol.)

# Algorithms

**Probability of a path through the model**

**Viterbi maximizes for seq**

**Forward sums of all possible paths**

**Forward Algorithm** – finds probability  $P$  that a model  $\lambda$  emits a given sequence  $O$  by summing over all paths that emit the sequence the probability of that path

**Viterbi Algorithm** – finds the most probable path through the model for a given sequence  
(both usually just boil down to simple applications of dynamic programming)